C usually is between 1/5 to 1/10. For most hydrocarbon mixtures, 1/7.5 is a good default value. The smaller the C constant, the slower the iteration scheme. However, if C is too large, the iteration will be oscillatory, and sometimes it might be divergent. The constant C should be adjusted to have good convergence. This method has been successfully applied to speed up iterative distillation calculation. It has also been successfully applied to calculate bubble point and dew point temperature under fixed pressure with a few cycles of iteration.

CONCLUSION

A new method, called the *hypothetical component method*, is presented as an alternative to the K_b method. This new method provides a simple way to estimate the bubble or dew point temperature for each stage and helps speed convergence of iterative distillation calculations.

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NOTATION

a_i, b_i = constants in Clausius-Clapeyron equation for i^{th} component
A, B = constants in Clausius-Clapeyron equation for the hypothetical component
C = constant
K_{ij} = vapor-liquid equilibrium constant of i^{th} component at j^{th} stage
NC = number of components
P = total pressure
P_v = vapor pressure
T = temperature
T_b = estimated bubble point temperature
T_d = estimated dew point temperature
T_j = temperature at j^{th} stage
$T_{j,n}$ = temperature at j^{th} stage during n^{th} trial
X_{ij} = liquid composition of i^{th} component at j^{th} stage
X_{if} = feed composition of i^{th} component
Y_{ij} = vapor composition of i^{th} component at j^{th} stage

Subscripts

f = feed
i = i^{th} component
j = j^{th} stage
n = n^{th} trial

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Extensional Viscosity and Recoil in Highly Dilute Polymer Solutions

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As one part of a current study of turbulent drag reduction with dilute polymer solutions, we have investigated the behavior of these materials in flow through an orifice. It is well known that for such a geometry many polymer melts and solutions adopt a "wine glass stem" appearance (Metzner, 1971; Bagley and Birks, 1960), consisting of a narrow rapidly converging central region, surrounded by large slowly recirculating eddies. Metzner, Uebler, and Chan Man Fong (1969) have analyzed the kinematics of this flow, arguing that large tensile stresses may arise within the converging section of the flow field. These stresses are reflected in the value of the pressure loss which may be significantly elevated relative to that of the solvent, even at extremely low polymer concentrations. Additional evidence of this effect is reported in the present note, along with a description of the recoil which occurs following sudden cessation of flow. This recoil effect offers a striking visual confirmation of the high extensional stresses which can develop in these systems.

EXPERIMENTAL

A 20 p.p.m. by weight solution of Separan AP 273 in deionized water was studied. Separan AP 273 is a polyacrylamide with a nominal molecular weight of 8 to 10 $\times$ 10^6, manufactured by Dow. The polymer has undergone approximately 30% hydrolysis and the resultant carboxyl groups converted to their sodium salt. Hence, in deionized water the solutions are slightly basic, with a pH of approximately 9 at 20 p.p.m. by weight. The flow setup consisted of a rectangular Plexiglas box, 30 cm long by 20 cm wide by 20 cm deep, fitted with a 0.35 cm long tube, 0.11 cm in diameter (hereafter referred to as the *orifice*). A distributor plate was located 23 cm upstream of the orifice. Pressure taps were mounted on the side walls, in the slowly recirculating region.

RESULTS AND DISCUSSION

Typical experimental data for the water and Separan solutions are tabulated in Table 1 vs. orifice velocity. The presence of the polymer leads to a marked elevation in

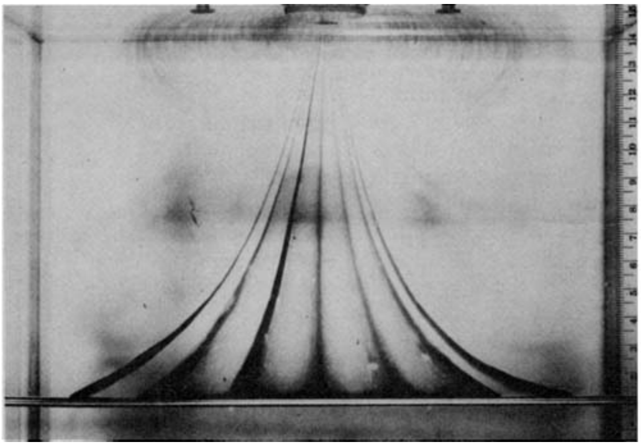

Fig. 1. Illustration of wine glass stem flow field for 20 p.p.m. by weight of Separan AP 273 in deionized water. Velocity, 770 cm/s. Orifice diameter, 0.11 cm. Scale on left calibrated in centimeters.

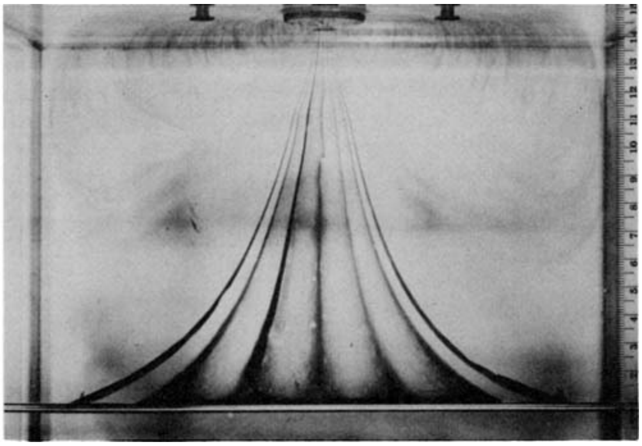

Fig. 2. Appearance of streamlines 3S. following sudden stoppage of flow in Figure 1.

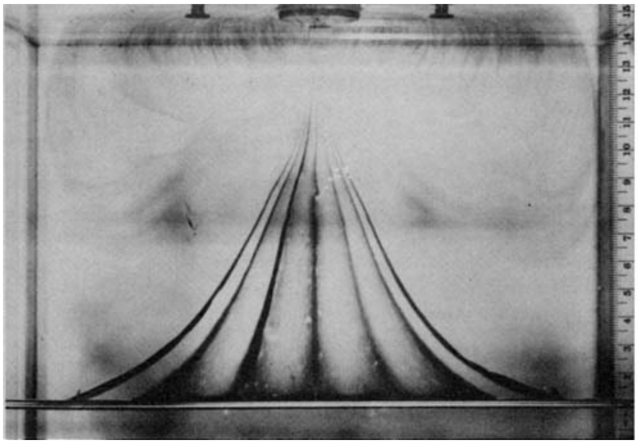

Fig. 3. Appearance of streamlines 6S. following sudden stoppage of flow in Figure 1.

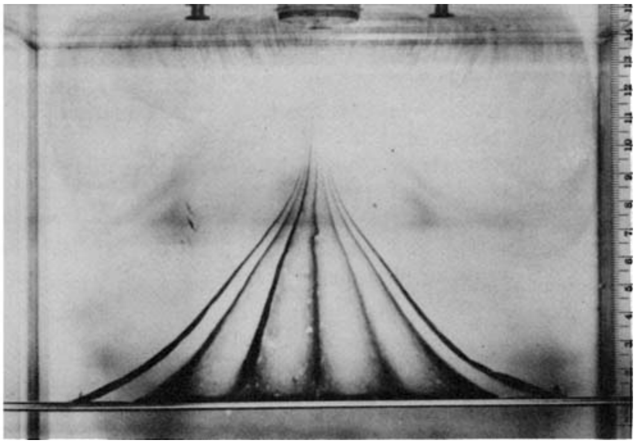

Fig. 4. Appearance of streamlines 9S. following sudden stoppage of flow in Figure 1.

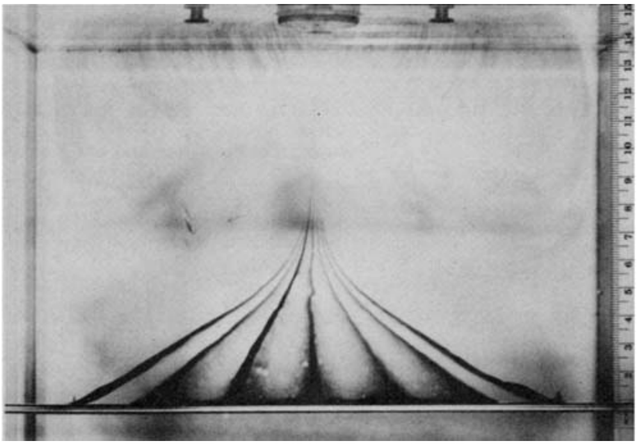

Fig. 5. Appearance of streamlines 15S. following sudden stoppage of flow in Figure 1.

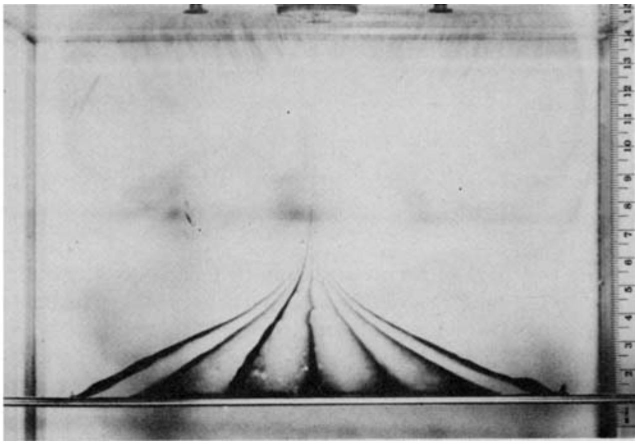

Fig. 6. Appearance of streamlines 21S. following sudden stoppage of flow in Figure 1. There was no further change in the streamlines after this time.

TABLE 1. PRESSURE LOSS DATA FOR WATER AND FOR POLYMER SOLUTION

Velocity (cm/s)	Pressure loss (N/m^2) Water	Pressure loss (N/m^2) Polymer	Convergence half angle (deg.)	Stretch rate (s^{-1})	Extensional viscosity (Ns/m^2)
490	1.0×10^4	2.0×10^4	14	3,800	5.3
620	1.7×10^4	2.7×10^4	22	7,300	3.7
930	3.9×10^4	5.9×10^4	18	9,200	6.4
1,100	6.1×10^4	8.0×10^4	18	11,000	7.3

pressure loss, presumably reflecting the high tensile stress in the converging flow field. To develop this relation more explicitly, we perform a momentum balance between the orifice entrance and a section of the emerging jet far enough downstream that all stresses are identically zero. This yields

$$\overline{T}_{11} = \rho v_o^2 \left[1 - \frac{R_o^2}{R_\infty^2} \right] - \overline{\tau}_w \frac{2L_o}{R_o} \quad (1)$$

where $\overline{T}_{11}$ is the average tensile stress at the orifice entrance, v_o the average velocity in the orifice, R_o and L_o the orifice radius and length, respectively, and R_∞ is the final radius of the emerging jet. $\overline{\tau}_w$ is the average wall shear stress existing within the orifice. Equation (1) assumes the velocity profile at the orifice entrance to be flat, a reasonable assumption in light of Figure 1 (see also Metzner, Uebler, and Chan Man Fong 1969; Metzner and Metzner 1970).

By following Metzner et al. (1969), $\overline{T}_{11}$ may be equated to the extensional normal stress difference $N_E = T_{11} - T_{22}$, minus the pressure in the recirculating region P_R:

$$T_{11} \cong N_E - P_R \quad (2)$$

Combining Equations (1) and (2), we get

$$N_E = P_R + \underbrace{\rho v_o^2 \left[1 - \frac{R_o^2}{R_\infty^2} \right]}_{\text{a.}} \underbrace{- \overline{\tau}_w \frac{2L_o}{R_o}}_{\text{b.}} \quad (3)$$

Visual observations as well as thrust measurements of the emerging jet indicate that term a. is negligible; R_o and R_∞ are equal within our limits of measurement. An upper bound on the magnitude of term b. may be obtained from the work of Collins and Schowalter (1963) on the entry region flow of Newtonian and power law fluids in tubes. Using their Figure 8, we find that this term is never more than about 20% of P_R in magnitude and decreases in relative importance with increasing velocity. It follows that to a good degree of approximation

$$N_E \cong P_R \quad (4)$$

The stretch rate or rate of extension at the orifice is given by (Metzner and Metzner, 1970)

$$d_{11} = \frac{Q \sin^3 \Phi}{\pi R_o^3 (1 - \cos \Phi)} \quad (5)$$

where Φ is the half angle of the converging region of fluid. Values of d_{11} are listed in Table 1. From Equations (4) and (5) we may obtain an expression for the extensional viscosity $\overline{\eta}$:

$$\overline{\eta} = \frac{N_E}{d_{11}} \quad (6)$$

Calculated values of $\overline{\eta}$ in Table 1 vary from 3.7 to 7.3 Ns/m^2, or approximately 1 500 to 3 000 times the shear viscosity! These values are similar in magnitude to those reported by Metzner and Metzner (1970) for a 100 p.p.m. by weight solution of Separan AP 30, a lower molecular weight polyacrylamide.

RECOIL

By suddenly plugging the orifice, we observed the behavior illustrated in Figures 1 to 6, a slow but very pronounced recoil, not seen at all with Newtonian fluids. This effect could also be observed at a concentration level of 10 p.p.m. by weight. To our knowledge, the only other published report of this effect is by Bagley and Birks (1960) on low-density polyethylene. This phenomenon offers dramatic qualitative support for the concept that high tensile stresses are present in the converging region.

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Problem Approximation for Stiff Ordinary Differential Equations

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The m dimensional set of ordinary differential equations

$$\frac{dz(t)}{dt} = \dot{\mathbf{z}} = \mathbf{F}(\mathbf{z}) \quad (1)$$

for which the local Jacobian $\partial \mathbf{F}/\partial \mathbf{z}$ has at least one eigenvalue that does not contribute significantly over most of the domain of interest, so-called *stiff systems*, presents severe requirements for stable and accurate numerical in-